Calculus 1 Workbook

Intermediate value theorem

krista king
MATH

INTERMEDIATE VALUE THEOREM

■ 1. Suppose the function $f(x)$ is continuous on the closed interval $[3,9]$ and that $f(3) = -4$ and $f(9) = 8$. The Intermediate Value Theorem guarantees that for some value c in the interval $[3,9]$, $f(c)$ will have a value in what interval?

■ 2. Suppose the function $g(x)$ is continuous on the closed interval $[0,12]$, and that $f(0) = 12$ and $f(12) = 2$. What is the domain of some value c if the Intermediate Value Theorem guarantees that for that value c, $f(c)$ will have a range of $[2,12]$?

■ 3. Suppose the function $h(x)$ is being discussed with some value c in regards to the Intermediate Value Theorem on the interval from a to b, where $b > a$. Select which conditions must be satisfied for $h(x)$ and the interval to apply to the Intermediate Value Theorem.

I - $f(c)$ must satisfy $f(a) \leq f(c) \leq f(b)$

II - $h(x)$ must be continuous and differentiable

III – The number c is $a < c < b$

IV - $h(x)$ must be continuous

V – The interval from a to b is an open interval

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VI – The number c is $a \leq c \leq b$

VII – The interval from a to b is a closed interval

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INTERMEDIATE VALUE THEOREM WITH AN INTERVAL

■ 1. The value $c = -1$ satisfies the conditions of the Intermediate Value Theorem for the function on the interval $[-3,5]$ because $f(c)$ equals what value?

$$f(x) = \frac{1}{4}(2x+5)(x-3)^2$$

■ 2. The value $c = 2$ does not satisfy the conditions of the Intermediate Value Theorem for $g(x) = 2x^2 - 11x + 4$ on the interval $[-2,8]$ because $g(c)$ equals what value?

■ 3. What value of c is guaranteed by the Intermediate Value Theorem on the interval $[-3,3]$ if $h(x) = 3(x+1)^3$ and $h(c) = 24$?

■ 4. What value of c is guaranteed by the Intermediate Value Theorem on the interval $[-5,6]$ if $f(c) = -6$ and

$$f(x) = \begin{cases} 3x - 10 & \text{if } x \leq 0 \\ x^2 + 3x - 10 & \text{if } 0 < x < 2 \\ 3x - 6 & \text{if } x \geq 2 \end{cases}$$

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■ 5. The value $c = 5$ satisfies the conditions of the Intermediate Value Theorem for the function on the interval $[3,9]$ because $g(c)$ equals what value?

$$g(x) = \frac{x^2 - 9}{x + 3}$$

■ 6. What value of c is guaranteed by the Intermediate Value Theorem on the interval $[3,6]$ if c is a root of $h(x)$.

$$h(x) = \frac{x^3 - 4x^2 - 11x + 30}{x^2 - 4}$$

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INTERMEDIATE VALUE THEOREM WITHOUT AN INTERVAL

■ 1. Use the Intermediate Value Theorem to prove that the equation $2e^x = 3\cos x$ has at least one positive solution. In what interval is that solution?

■ 2. Use the Intermediate Value Theorem to prove that the equation $3\sin x + 7 = x^2 - 2x - 2$ has at least one positive solution. In what interval is that solution?

■ 3. Use the Intermediate Value Theorem to prove that the equation $x^6 - 9x^4 + 7 = x^5 - 8x^3 - 9$ has at least one positive solution. In what interval is that solution?

■ 4. Use the Intermediate Value Theorem to prove that the equation $4e^{x-3} = -2(x^3 - 5x + 9)$ has at least one negative solution. In what interval is that solution?

■ 5. Use the Intermediate Value Theorem to show that the equation has at least one positive solution. In what interval is that solution?

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$$6e^{-x} = -\left(\frac{1}{5}x^2 - 4x + 9\right)$$

■ 6. Use the Intermediate Value Theorem to show that the equation $2\sin(4x - 1) = \cos(2x - 3)$ has at least one negative solution. In what interval is that solution?

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